The Natural Logarithm

$\ln x$, $x > 0$. The natural logarithm. The log of x to the base e (e = 27182818........)

If $\ln x = y$, then $e^y = x$.

$\ln 1 = 0$ and $\ln e = 1$. $\ln x$ is the inverse function of e^x and so its graph will be the reflection in the line $y = x$ of the graph of e^x.

Some useful properties of the natural log, derived from general log laws:

$1.\ \ln(A\times B)=\ln A+\ln B$

$2.\ \ln\left(\frac{A}{B}\right)=\ln A-\ln B$

$3.\ \ln x^r=r\ln x$ for any real number r

$4.\ \log_a x=\frac{\ln x}{\ln a}$ for any base $a > 0$ except 1

$5.\ \ln x=\frac{\log_a x}{\log_a e}$ any base $a > 0$ except 1

$6.\ \ln e^x=x$

$7.\ e^{\ln x}=x$

Now, let's determine the derivative of $\ln x$. To do this, we will make use of two equations, these being $e^{\ln x}=x$ (1) and $\frac{d}{dx}e^{f(x)}=f'(x)e^{f(x)}$ (2).

Now, $\frac{d}{dx}e^{\ln x}=\frac{d}{dx}x=1$

Using equation (2), we get $\frac{d}{dx}e^{\ln x}=\frac{d}{dx}\ln x\times e^{lnx}=1$

So, $\frac{d}{dx}\ln x\times x=1$, making $\frac{d}{dx}\ln x=\frac{1}{x}$.

Here's an alternative method and it makes use of implicit differentiation.

Let $y = \ln x$, then

$e^y = x$ (3)

Now apply implicit differentiation to (3) and we get

$e^y\frac{dy}{dx}=1$

So, $\frac{dy}{dx}=\frac{1}{e^y}$

Since $e^y = x$, we get $\frac{dy}{dx} = \frac{1}{x}$

This will now also mean that $\int \frac{1}{x} dx = \ln x + C$ and $\int_a^b \frac{1}{x} dx = \ln b - \ln a = \ln\left(\frac{b}{a}\right)$

Can you identify the place in the photo below?